Section 11.1 Vectors in the Plane

- Write the component form of a vector.
- Perform vector operations and interpret the results geometrically.
- Write a vector as a linear combination of standard unit vectors.
- Use vectors to solve problems involving force or velocity.

Component Form of a Vector

Many quantities in geometry and physics, such as area, volume, temperature, and time, can be characterized by a single real number scaled to appropriate measure. These are called scalar quantities, and the real number associated with them is called a scalar.

Other quantities, such as force, velocity, and acceleration, involve both magnitude and direction and cannot be characterized completely by a single real number. A directed line segment is used to represent such a quantity, as shown in Figure 11.1. The directed line segment \( \overrightarrow{PQ} \) has initial point \( P \) and terminal point \( Q \), and its length (or magnitude) is denoted by \( || \overrightarrow{PQ} || \). Directed line segments that have the same magnitude and direction are equivalent, as shown in Figure 11.2. The set of all directed line segments that are equivalent to a given directed line segment \( \overrightarrow{PQ} \) is a vector plane and is denoted by \( \mathbf{v} = \overrightarrow{PQ} \). In typeset material, vectors are usually denoted by lowercase, boldface letters such as \( \mathbf{u}, \mathbf{v}, \) and \( \mathbf{w} \). When written by hand, vectors are often denoted by letters with arrows above them, such as \( \vec{u}, \vec{v}, \vec{w} \).

Be sure you see that a vector in the plane can be represented by many different directed line segments.
CHAPTER 11  Vectors and the Geometry of Space

Section 11.1  Vectors in the Plane

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Component Form of a Vector

Many quantities in geometry and physics, such as area, volume, temperature, mass, and time, can be characterized by a single real number scaled to appropriate units of measure. These are called scalar quantities, and the real number associated with each is called a scalar.

Other quantities, such as force, velocity, and acceleration, involve both magnitude and direction and cannot be characterized completely by a single real number. A directed line segment is used to represent such a quantity, as shown in Figure 11.1. The directed line segment \( \overrightarrow{PQ} \) has initial point \( P \) and terminal point \( Q \), and its length (or magnitude) is denoted by \( ||PQ|| \). Directed line segments that have the same length and direction are equivalent, as shown in Figure 11.2. The set of all directed line segments that are equivalent to a given directed line segment \( \overrightarrow{PQ} \) is a vector in the plane and is denoted by \( \mathbf{v} = \overrightarrow{PQ} \). In typeset material, vectors are usually denoted by lowercase, boldface letters such as \( \mathbf{u}, \mathbf{v}, \) and \( \mathbf{w} \). When written by hand, however, vectors are often denoted by letters with arrows above them, such as \( \overrightarrow{u}, \overrightarrow{v}, \) and \( \overrightarrow{w} \).

Be sure you see that a vector in the plane can be represented by many different
Component Form of a Vector

Many quantities in geometry and physics, such as area, volume, temperature, and time, can be characterized by a single real number scaled to appropriate measure. These are called scalar quantities, and the real number associated is called a scalar.

Other quantities, such as force, velocity, and acceleration, involve both magnitude and direction and cannot be characterized completely by a single real number. A directed line segment is used to represent such a quantity, as shown in Figure 11.1. The directed line segment \( \overrightarrow{PQ} \) has initial point \( P \) and terminal point \( Q \), and (or magnitude) is denoted by \( \| \overrightarrow{PQ} \| \). Directed line segments that have the same magnitude and direction are equivalent, as shown in Figure 11.2. The set of all directed line segments that are equivalent to a given directed line segment \( \overrightarrow{PQ} \) is a vector, and is denoted by \( \mathbf{v} = \overrightarrow{PQ} \). In typeset material, vectors are usually denoted by lowercase, boldface letters such as \( \mathbf{u} \), \( \mathbf{v} \), and \( \mathbf{w} \). When written by hand, vectors are often denoted by letters with arrows above them, such as \( \mathbf{u} \) and \( \mathbf{v} \).

Be sure you see that a vector in the plane can be represented by many directed line segments—all pointing in the same direction and all of the same length.

**EXAMPLE 1** Vector Representation by Directed Line Segments

Let \( \mathbf{v} \) be represented by the directed line segment from \((0,0)\) to \((3,2)\), and \( \mathbf{v} \) represented by the directed line segment from \((1,2)\) to \((4,4)\). Show that \( \mathbf{v} \) is equivalent.

**Solution** Let \( P(0,0) \) and \( Q(3,2) \) be the initial and terminal points of \( \overrightarrow{PQ} \), and let \( S(1,2) \) and \( T(4,4) \) be the initial and terminal points of \( \overrightarrow{ST} \), as shown in Figure 11.2. Then

\[
\overrightarrow{PQ} = \overrightarrow{ST} = \mathbf{v}.
\]
Try It One

Section 11.1

\( P(9, 12) \) to \( Q(24, 9) \)

\( R(-5, 2) \) to \( S(10, -5) \)

Show Equivalent

\[ \| PQ \| = \sqrt{(24-9)^2 + (9-12)^2} = \sqrt{15^2 + (-3)^2} = \sqrt{234} \]

\[ \| RS \| = \sqrt{(10-(-5))^2 + (-5-2)^2} = \sqrt{15^2 + 63^2} = \sqrt{234} \]

Same Length

\[ \text{Slope}_{\overrightarrow{PQ}} = \frac{9-12}{24-9} = \frac{-3}{15} = -\frac{1}{5} \]

\[ \text{Slope}_{\overrightarrow{RS}} = \frac{-5-(-2)}{10-(-5)} = \frac{-3}{15} = -\frac{1}{5} \]

Vectors Are Equivalent!
represented by the directed line segment from \((-5, -2)\) to \((10, -5)\). Show that \(\mathbf{v}\) and \(\mathbf{u}\) are equivalent.

**Solution**

Let \(P(0, 12)\) and \(Q(24, 9)\) be the initial and terminal points of \(\mathbf{v}\), and let \(R(-5, -3)\) and \(S(10, -5)\) be the initial and terminal points of \(\mathbf{u}\), as shown in the figure. You can use the Distance Formula to show that \(PQ\) and \(RS\) have the same length.

\[
|PQ| = \sqrt{(24 - 9)^2 + (9 - 12)^2} = \sqrt{234} \quad \text{Length of } PQ
\]

\[
|RS| = \sqrt{(10 - (-5))^2 + (-5 - (-2))^2} = \sqrt{234} \quad \text{Length of } RS
\]

Both line segments have the same direction, because they both are directed toward the lower right on lines having the same slope.

Slope of \(PQ\) is \(\frac{9 - 12}{24 - 9} = \frac{-3}{15} = \frac{-1}{5}\)

and

Slope of \(RS\) is \(\frac{-5 - (-2)}{10 - (-5)} = \frac{-3}{15} = \frac{-1}{5}\)

Because \(PQ\) and \(RS\) have the same length and direction, you can conclude that the two vectors are equivalent. That is, \(\mathbf{v}\) and \(\mathbf{u}\) are equivalent.
SECTION 11.1 Vectors in the Plane

The directed line segment whose initial point is the origin is often the convenient representative of a set of equivalent directed line segments such as shown in Figure 11.3. This representation of \( \mathbf{v} \) is said to be in **standard position** if the directed line segment whose initial point is the origin can be uniquely represented by the coordinates of its terminal point \( Q(v_1, v_2) \), as shown in Figure 11.4.

**Definition of Component Form of a Vector in the Plane**

If \( \mathbf{v} \) is a vector in the plane whose initial point is the origin and whose terminal point is \((v_1, v_2)\), then the **component form** of \( \mathbf{v} \) is given by

\[
\mathbf{v} = (v_1, v_2)
\]

The coordinates \( v_1 \) and \( v_2 \) are called the **components** of \( \mathbf{v} \). If both the initial point and the terminal point lie at the origin, then \( \mathbf{v} \) is called the **zero vector** and is denoted by \( \mathbf{0} = (0, 0) \).

This definition implies that two vectors \( \mathbf{u} = (u_1, u_2) \) and \( \mathbf{v} = (v_1, v_2) \) are equal only if \( u_1 = v_1 \) and \( u_2 = v_2 \).

The following procedures can be used to convert directed line segments into component form or vice versa.

1. If \( P(p_1, p_2) \) and \( Q(q_1, q_2) \) are the initial and terminal points of a directed segment, the component form of the vector \( \mathbf{v} \) represented by \( \overrightarrow{PQ} \) is \((q_1 - p_1, q_2 - p_2)\).
The coordinates \( v_1 \) and \( v_2 \) are called the components of \( \mathbf{v} \) if both the initial point and the terminal point lie at the origin, then \( \mathbf{v} \) is called the zero vector and is denoted by \( \mathbf{0} = (0, 0) \).

This definition implies that two vectors \( \mathbf{u} = (u_1, u_2) \) and \( \mathbf{v} = (v_1, v_2) \) are equal if and only if \( u_1 = v_1 \) and \( u_2 = v_2 \).

The following procedures can be used to convert directed line segments to component form or vice versa.

1. If \( P(p_1, p_2) \) and \( Q(q_1, q_2) \) are the initial and terminal points of a directed line segment, the component form of the vector \( \mathbf{v} \) represented by \( \overrightarrow{PQ} \) is \( (v_1, v_2) = (q_1 - p_1, q_2 - p_2) \). Moreover, the length (or magnitude) of \( \mathbf{v} \) is

\[
\| \mathbf{v} \| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2}
\]

Length of a vector

2. If \( \mathbf{v} = (v_1, v_2) \), \( \mathbf{v} \) can be represented by the directed line segment, in standard position, from \( P(0, 0) \) to \( Q(v_1, v_2) \).

The length of \( \mathbf{v} \) is also called the norm of \( \mathbf{v} \). If \( \| \mathbf{v} \| = 1 \), \( \mathbf{v} \) is a unit vector. Moreover, \( \| \mathbf{v} \| = 0 \) if and only if \( \mathbf{v} \) is the zero vector \( \mathbf{0} \).

**Example 2** Finding the Component Form and Length of a Vector

Find the component form and length of the vector \( \mathbf{v} \) that has initial point \((3, -7)\) and terminal point \((-2, 5)\).

**Solution** Let \( P(3, -7) = (p_1, p_2) \) and \( Q(-2, 5) = (q_1, q_2) \). Then the components
Try It 2

\[ \overrightarrow{PQ} \text{ Initial to Terminal} \]

Terminal - Initial

\[ \overrightarrow{PQ} = (y-3, y-(-5)) = (1, 12) \]

Component Form

\[ || \overrightarrow{PQ} || = \sqrt{(y-3)^2 + (y-(-5))^2} = \sqrt{y^2 + 12^2} = \sqrt{145} \]

\[ (3, 5) \]
Vector Operations

Definitions of Vector Addition and Scalar Multiplication

Let \( \mathbf{u} = (u_1, u_2) \) and \( \mathbf{v} = (v_1, v_2) \) be vectors and let \( c \) be a scalar.

1. The vector sum of \( \mathbf{u} \) and \( \mathbf{v} \) is the vector \( \mathbf{u} + \mathbf{v} = (u_1 + v_1, u_2 + v_2) \).
2. The scalar multiple of \( c \) and \( \mathbf{u} \) is the vector \( c\mathbf{u} = (cu_1, cu_2) \).
3. The negative of \( \mathbf{v} \) is the vector \( -\mathbf{v} = (-1)\mathbf{v} = (-v_1, -v_2) \).
4. The difference of \( \mathbf{u} \) and \( \mathbf{v} \) is \( \mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v}) = (u_1 - v_1, u_2 - v_2) \).

Geometrically, the scalar multiple of a vector \( \mathbf{v} \) and a scalar \( c \) is the vector that is \( |c| \) times as long as \( \mathbf{v} \), as shown in Figure 11.6. If \( c \) is positive, \( c\mathbf{v} \) has the same direction as \( \mathbf{v} \). If \( c \) is negative, \( c\mathbf{v} \) has the opposite direction.

The sum of two vectors can be represented geometrically by positioning the vectors (without changing their magnitudes or directions) so that the initial point of one coincides with the terminal point of the other, as shown in Figure 11.7. The vector \( \mathbf{u} + \mathbf{v} \), called the resultant vector, is the diagonal of a parallelogram having \( \mathbf{u} \) and \( \mathbf{v} \) as its adjacent sides.
Geometrically, the scalar multiple of a vector \(v\) and a scalar \(c\) is the vector that is \([c]\) times as long as \(v\), as shown in Figure 11.6. If \(c\) is positive, \(cv\) has the same direction as \(v\). If \(c\) is negative, \(cv\) has the opposite direction.

The sum of two vectors can be represented geometrically by positioning the vectors (without changing their magnitudes or directions) so that the initial point of one coincides with the terminal point of the other, as shown in Figure 11.7. The vector \(u + v\), called the resultant vector, is the diagonal of a parallelogram having \(u\) and \(v\) as its adjacent sides.

To find \(u + v\),

1. Move the initial point of \(v\) to the terminal point of \(u\), or
2. Move the initial point of \(u\) to the terminal point of \(v\).

Figure 11.7

**Animation**

Figure 11.8 shows the equivalence of the geometric and algebraic definitions of vector addition and scalar multiplication, and presents (at far right) a geometric interpretation of \(u - v\).
To find $u + v$.

(1) move the initial point of $v$ to the terminal point of $u$, or
(2) move the initial point of $u$ to the terminal point of $v$.

Figure 11.7

Animation

Figure 11.8 shows the equivalence of the geometric and algebraic definitions of vector addition and scalar multiplication, and presents (at far right) a geometric interpretation of $u - v$.

Vector addition

Scalar multiplication

Vector subtraction $u - v$
Try It 3

\[ \mathbf{v} = \langle 3, 20 \rangle \quad \mathbf{w} = \langle -2, -9 \rangle \]

a) \[ \frac{2}{3} \mathbf{v} = \frac{2}{3} \langle 3, 20 \rangle = \langle 2, \frac{40}{3} \rangle \]

Multiply By A Scalar

b) \[ \mathbf{w} - \mathbf{v} = \langle -2, -9 \rangle - \langle 3, 20 \rangle \]
\[ = \langle -2 - 3, -9 - 20 \rangle = \langle -5, -29 \rangle \]

c) \[ 2 \mathbf{w} + 5 \mathbf{v} = 2 \langle -2, -9 \rangle + 5 \langle 3, 20 \rangle \]
\[ = \langle -4, -18 \rangle + \langle 15, 100 \rangle \]
\[ = \langle 11, 82 \rangle \]
Exploration 3B

\[ \mathbf{A} = \mathbf{H} - \mathbf{G} \]
\[ \mathbf{A} = -\mathbf{G} + \mathbf{H} \]
\[ \mathbf{E} = \mathbf{A} + \mathbf{B} + \mathbf{C} - \mathbf{D} \]
\[ \mathbf{E} = -\mathbf{B} + \mathbf{A} + \mathbf{C} + \mathbf{D} \]
Vector addition and scalar multiplication share many properties of ordinary arithmetic, as shown in the following theorem.

**THEOREM 11.1 Properties of Vector Operations**

Let \( u, v, \) and \( w \) be vectors in the plane, and let \( c \) and \( d \) be scalars.

1. \( u + v = v + u \) (Commutative Property)
2. \( (u + v) + w = u + (v + w) \) (Associative Property)
3. \( u + 0 = u \) (Additive Identity Property)
4. \( u + (-u) = 0 \) (Additive Inverse Property)
5. \( c(du) = (cd)u \) (Distributive Property)
6. \( (c + d)u = cu + du \) (Distributive Property)
7. \( c(u + v) = cu + cv \) (Distributive Property)
8. \( 1(u) = u, 0(u) = 0 \) (Distributive Property)

**Proof** The proof of the Associative Property of vector addition uses the Associative Property of addition of real numbers.

\[
(u + v) + w = [(u_1, u_2) + (v_1, v_2)] + (w_1, w_2)
= (u_1 + v_1, u_2 + v_2) + (w_1, w_2)
\]
CHAPTER 11  Vectors and the Geometry of Space

Any set of vectors (with an accompanying set of scalars) that satisfies the eight properties given in Theorem 11.1 is a **vector space**. The eight properties are the **vector space axioms**. So, this theorem states that the set of vectors in the plane (with the set of real numbers) forms a vector space.

**THEOREM 11.2  Length of a Scalar Multiple**

Let \( v \) be a vector and let \( c \) be a scalar. Then

\[
\|cv\| = |c| \|v\|. \quad \text{[} |c| \text{ is the absolute value of } c.\text{]}
\]

**Proof**  Because \( cv = (cv_1, cv_2) \), it follows that

\[
\|cv\| = \|(cv_1, cv_2)\| = \sqrt{(cv_1)^2 + (cv_2)^2} = \sqrt{c^2v_1^2 + c^2v_2^2} = \sqrt{c^2(v_1^2 + v_2^2)} = |c| \sqrt{v_1^2 + v_2^2} = |c| \|v\|.
\]

In many applications of vectors, it is useful to find a unit vector that has the same direction as a given vector. The following theorem gives a procedure for doing this.
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**THEOREM 11.3 Unit Vector in the Direction of \( \mathbf{v} \)**

If \( \mathbf{v} \) is a nonzero vector in the plane, then the vector

\[
\mathbf{u} = \left( \frac{1}{\|\mathbf{v}\|} \right) \mathbf{v}
\]

has length 1 and the same direction as \( \mathbf{v} \).

**Proof**  Because \( 1/\|\mathbf{v}\| \) is positive and \( \mathbf{u} = (1/\|\mathbf{v}\|) \mathbf{v} \), you can conclude that \( \mathbf{u} \) has the same direction as \( \mathbf{v} \). To see that \( \|\mathbf{u}\| = 1 \), note that

\[
\|\mathbf{u}\| = \left\| \left( \frac{1}{\|\mathbf{v}\|} \right) \mathbf{v} \right\|
= \frac{1}{\|\mathbf{v}\|} \|\mathbf{v}\|
= \frac{1}{\|\mathbf{v}\|} \|\mathbf{v}\|
= 1.
\]

So, \( \mathbf{u} \) has length 1 and the same direction as \( \mathbf{v} \).
Try It!  \[ \mathbf{v} = \langle 8, -6 \rangle \]

\[ \|\mathbf{v}\| = \sqrt{(8)^2 + (-6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10 \]

Unit Vector

\[ \frac{1}{10} \cdot \langle 8, -6 \rangle = \langle \frac{8}{10}, \frac{-6}{10} \rangle \]

Unit Vector Length

\[ \|\text{Unit Vector}\| = \sqrt{(\frac{8}{10})^2 + (\frac{-6}{10})^2} = \sqrt{\frac{64}{100} + \frac{36}{100}} = \sqrt{\frac{100}{100}} = 1 \]
This vector has length 1, because
\[
\sqrt{\frac{-2}{29}} + \frac{5}{\sqrt{29}} = \sqrt{\frac{4}{29} + \frac{25}{29}} = \sqrt{\frac{29}{29}} = 1.
\]

**Try It**

**Exploration A**

Generally, the length of the sum of two vectors is not equal to the sum of their lengths. To see this, consider the vectors \( u \) and \( v \) as shown in Figure 11.9. By considering \( u \) and \( v \) as two sides of a triangle, you can see that the length of the third side is \( \| u + v \| \), and you have
\[
\| u + v \| \leq \| u \| + \| v \|.
\]

Equality occurs only if the vectors \( u \) and \( v \) have the same direction. This result is called the **triangle inequality** for vectors. (You are asked to prove this in Exercise 89, Section 11.3.)

**Standard Unit Vectors**

The unit vectors \((1, 0)\) and \((0, 1)\) are called the **standard unit vectors** in the plane and are denoted by
\[
i = (1, 0) \quad \text{and} \quad j = (0, 1)
\]
as shown in Figure 11.10. These vectors can be used to represent any vector uniquely, as follows.
\[
v = (v_1, v_2) = (v_1, 0) + (0, v_2) = v_1(1, 0) + v_2(0, 1) = v_1i + v_2j
\]
**Component Form**

\[ \langle 3, 7 \rangle \]

**Standard Unit Vectors**

The unit vectors \((1, 0)\) and \((0, 1)\) are called the **standard unit vectors** and are denoted by:

\[ i = \langle 1, 0 \rangle \quad \text{and} \quad j = \langle 0, 1 \rangle \]

as shown in Figure 11.10. These vectors can be used to represent any vector \( \mathbf{u} \) as follows.

\[ \mathbf{v} = (v_1, v_2) = (v_1, 0) + (0, v_2) = v_1 (1, 0) + v_2 (0, 1) = v_1 i + v_2 j \]

The vector \( \mathbf{v} = v_1 i + v_2 j \) is called a **linear combination** of \( i \) and \( j \). The scalars \( v_1 \) and \( v_2 \) are called the **horizontal and vertical components** of \( \mathbf{v} \).

**EXAMPLE 5 Writing a Linear Combination of Unit Vectors**

Let \( \mathbf{u} \) be the vector with initial point \((2, -5)\) and terminal point \((-1, 3)\).

\[ \mathbf{v} = 2i - j \text{.} \]

Write each vector as a linear combination of \( i \) and \( j \).

**Solution**

\[ a. \quad \mathbf{u} = (q_1 - p_1, q_2 - p_2) = (2 - 2, -5 - 3) = (-1, -5) = -1i - 5j \]

\[ b. \quad \mathbf{w} = 2\mathbf{u} - 3\mathbf{v} = 2(-1i - 5j) - 3(2i - j) = -2i - 10j - 6i + 3j = -8i - 7j \]
Try It 5

V = \mathbf{u} - 2 \mathbf{w}

**Standard Unit Form**

\[ \mathbf{u} = 2\mathbf{i} - 1\mathbf{j} \]

\[ \mathbf{w} = 1\mathbf{i} + 2\mathbf{j} \]

\[ V = \mathbf{u} - 2\mathbf{w} = (2\mathbf{i} - 1\mathbf{j}) - 2(1\mathbf{i} + 2\mathbf{j}) \]

\[ = (2\mathbf{i} - 1\mathbf{j}) + (-2\mathbf{i} - 4\mathbf{j}) \]

\[ \mathbf{V} = 0\mathbf{i} - 5\mathbf{j} = \langle 0, -5 \rangle \]
CHAPTER 11  Vectors and the Geometry of Space

If \( \mathbf{u} \) is a unit vector and \( \theta \) is the angle (measured counterclockwise) from the positive x-axis to \( \mathbf{u} \), then the terminal point of \( \mathbf{u} \) lies on the unit circle, and you have

\[
\mathbf{u} = (\cos \theta, \sin \theta) = \cos \theta \mathbf{i} + \sin \theta \mathbf{j}
\]

as shown in Figure 11.11. Moreover, it follows that any other nonzero vector \( \mathbf{v} \) making an angle \( \theta \) with the positive x-axis has the same direction as \( \mathbf{u} \), and you can write

\[
\mathbf{v} = ||\mathbf{v}|| (\cos \theta, \sin \theta) = ||\mathbf{v}|| \cos \theta \mathbf{i} + ||\mathbf{v}|| \sin \theta \mathbf{j}.
\]

**EXAMPLE 6**  Writing a Vector of Given Magnitude and Direction

The vector \( \mathbf{v} \) has a magnitude of 3 and makes an angle of \( 30^\circ = \pi/6 \) with the positive x-axis. Write \( \mathbf{v} \) as a linear combination of the unit vectors \( \mathbf{i} \) and \( \mathbf{j} \).

**Solution**  Because the angle between \( \mathbf{v} \) and the positive x-axis is \( \theta = \pi/6 \), you can write the following.

\[
\mathbf{v} = ||\mathbf{v}|| \cos \theta \mathbf{i} + ||\mathbf{v}|| \sin \theta \mathbf{j} = 3 \cos \frac{\pi}{6} \mathbf{i} + 3 \sin \frac{\pi}{6} \mathbf{j}
\]
Try It 6

\[ \vec{V} = \]

\[ \| \vec{V} \| = 1 \]

\[ \Theta = 45^\circ = \frac{\pi}{4} \]

\[ V = \| \text{null} \| \cos \Theta \hat{i} + \| \text{null} \| \sin \Theta \hat{j} \]

\[ V = 1 \cdot \cos \left( \frac{\pi}{4} \right) \hat{i} + 1 \cdot \sin \left( \frac{\pi}{4} \right) \hat{j} \]

\[ V = \frac{\sqrt{2}}{2} \hat{i} + \frac{\sqrt{2}}{2} \hat{j} \]

Length = 1 unit
The problem involves finding the resultant force \( \mathbf{F} \) given three forces \( \mathbf{F}_1 \), \( \mathbf{F}_2 \), and \( \mathbf{F}_3 \). The forces are represented in component form.

1. \( \mathbf{F}_1 = 300 \langle \cos(30^\circ), \sin(20^\circ) \rangle \)
   - \( \mathbf{F}_1 = 300 \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle \)
   - \( \mathbf{F}_1 = \langle 150\sqrt{3}, -150 \rangle \)

2. \( \mathbf{F}_2 = 180 \langle \cos(45^\circ), \sin(45^\circ) \rangle \)
   - \( \mathbf{F}_2 = 180 \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle = \langle 90\sqrt{2}, 90 \sqrt{2} \rangle \)

3. \( \mathbf{F}_3 = 250 \langle \cos(135^\circ), \sin(135^\circ) \rangle = 250 \langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle = \langle -125\sqrt{2}, 125 \sqrt{2} \rangle \)

The resultant force \( \mathbf{F} \) is the sum of the individual forces:

\[
\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \langle 150\sqrt{3} + 90\sqrt{2} - 125\sqrt{2}, -150 + 90\sqrt{2} + 125 \rangle
\]

\[
= \langle 150\sqrt{3} - 35\sqrt{2}, -150 + 215 \sqrt{2} \rangle
\]

\[
= (150\sqrt{3} - 35\sqrt{2}) \mathbf{i} + (-150 + 215 \sqrt{2}) \mathbf{j}
\]

\[
\mathbf{F} = 266.7 \mathbf{i} + 159.1 \mathbf{j}
\]

The magnitude of the resultant force is:

\[
\text{Magnitude} = \sqrt{(266.7)^2 + (159.1)^2} = 260.7 \text{ Newtons}
\]

The direction of the resultant force can be found using the components:

\[
\cos \theta = \frac{266.7}{260.7}, \quad \sin \theta = \frac{159.1}{260.7}
\]

\[
\theta = \cos^{-1} \left( \frac{266.7}{260.7} \right) = 36.2^\circ = 0.63 \text{ radians}
\]

\[
\mathbf{F} = 266.7 \langle \cos 36.2^\circ, \sin 36.2^\circ \rangle
\]
\[ V = V_1 + V_2 \]

\[ V_1 = 450 \hat{i} \]

\[ V_2 = 50 \cos(135^\circ) \hat{i} + 50 \sin(135^\circ) \]