Section 3.2
Conditional Probability and the Multiplication Rule

The multiplication rules can be used to find the probability of two or more events that occur in sequence.

**Conditional probability** is the probability of an event occurring, given that another event has already occurred. The conditional probability of event B occurring, given that event A has already occurred is denoted by:

\[ P(B|A) \quad \text{read as "the probability of B given A"} \]

**Independent Events**

Two events A and B are **independent events** if the fact that A occurs does not affect the probability of B occurring. When two events are independent, the probability of both occurring is:

\[ P(A \text{ and } B) = P(A) \cdot P(B) \]

**Example:** Flip a coin then roll a die.

\[ P(\text{Heads and Roll 1}) = P(\text{Heads}) \cdot P(\text{Roll 1}) \]

\[ = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12} \]

**Dependent Events**

Two events are said to be **dependent events** if the occurrence of the first event affects the occurrence of the second event in such a way that the probability is changed.

When two events are dependent, the probability of both occurring is

\[ P(A \text{ and } B) = P(A) \cdot P(B|A) \]

**Example:** There are 9 batteries in a drawer; 5 are dead and 4 are good. Find the probability of randomly selecting 2 good batteries to put in a flashlight.

\[ P(\text{Good and Good}) = \frac{5}{9} \cdot \frac{4}{8} = \frac{5}{18} = .278 \]

Find the probability of randomly selecting 3 dead batteries to put in a flashlight.

\[ P(D, D, D) = \frac{4}{9} \cdot \frac{3}{8} \cdot \frac{2}{7} = \frac{1}{21} = .048 \]

The key word **AND** tells us to **MULTIPLY** probabilities.

**Examples with Independent and Dependent Events**
What is the probability that a randomly selected classmate used a seat belt the last time they got into a car?

\[ \frac{20}{37} = 0.541 \]

What is the probability that 4 randomly selected classmates used a seat belt?

Without Replacement:

\[ \frac{20}{37} \cdot \frac{19}{36} \cdot \frac{18}{35} \cdot \frac{17}{34} = 0.276 \]

With Replacement:

\[ \frac{20}{27} \cdot \frac{20}{27} \cdot \frac{20}{27} \cdot \frac{20}{27} = 0.301 \]

For large populations and small samples (no more than 5%) assume replacement.

The Gallup Poll reported that 52% of Americans used a seat belt the last time they got into a car. If four Americans are selected at random, find the probability that all used a seat belt the last time they got into a car.

\[ P(\text{all 4}) = 0.52 \cdot 0.52 \cdot 0.52 \cdot 0.52 = 0.073 \]

Additional questions:

Find the probability that a person did not use a seat belt.

\[ P(\text{used seatbelt}) = 0.52 \quad P(\text{not use seatbelt}) = 0.48 \]

If four people are selected at random, find the probability that none of them used a seat belt.

\[ P(\text{none of 4}) = (0.48)(0.48)(0.48)(0.48) = 0.053 \]

If 2 cards are selected from a standard deck of 52 cards without replacement, find these probabilities.

a) Both are spades.

\[ P(\text{spade and spade}) = \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17} \]

b) Both are the same suit.

\[ P(\text{suit and same suit}) = \frac{13}{52} \cdot \frac{12}{51} = \frac{4}{17} \]

c) Both are kings.

\[ P(K \text{ and } K) = \frac{4}{13} \cdot \frac{3}{51} = \frac{1}{66} \]

\[ P(A \text{ and } B) = P(A) - P(B|A) \]
Section 3.2 Continued

Probabilities for “At Least One”

Use for problems like #21

The complement of “at least one” is “none”, so

\[ P(\text{at least one}) = 1 - P(\text{none}) \]

Example:
The Gallup Poll reported that 52% of Americans used a seat belt the last time they got into a car. If four people are selected at random, find the probability that at least one of them used a seat belt.

\[ \text{So 48\% did not use a seat belt} \]

\[ \text{So to find} \]

\[ P(\text{at least 1 used belt}) = 1 - P(\text{none used belt}) \]

\[ = 1 - (0.48)^4 \]

\[ = 0.947 \]

Example:
A lot of 18 portable radios contains 3 defective ones. Two are selected without replacement and tested.

Independent events

a) Find the probability both are not defective.

\[ P(\text{not def, not def}) = \frac{15}{18} \cdot \frac{14}{17} = 0.686 \]

b) Find the probability both are defective.

\[ P(\text{def, def}) = \frac{3}{18} \cdot \frac{2}{17} = 0.020 \]

c) Find the probability that at least one will be defective.

\[ P(\text{at least 1 def}) = 1 - P(\text{none are defective}) \]

\[ = 1 - 0.686 \]

\[ = 0.314 \]
Conditional Probability

Example:
A blood bank catalogs the types of blood given by donors during the last five days.

<table>
<thead>
<tr>
<th>Type O</th>
<th>Type A</th>
<th>Type B</th>
<th>Type AB</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rh-positive</td>
<td>156</td>
<td>139</td>
<td>37</td>
<td>12</td>
</tr>
<tr>
<td>Rh-Negative</td>
<td>28</td>
<td>25</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>184</td>
<td>164</td>
<td>45</td>
<td>16</td>
</tr>
</tbody>
</table>

A donor is selected at random.
Find the probability the donor is Rh-negative given they have type O blood.

\[ P(Rh^- | O) = \frac{28}{184} \approx 0.152 \]

Find the probability the donor has type B given they are Rh-positive.

\[ P(B | Rh^+) = \frac{37}{344} = 0.108 \]

The Multiplication Rule and Conditional Probability

Solve \( P(A \text{ and } B) = P(A) \cdot P(B | A) \) for \( P(B | A) \).

The conditional probability of event B occurring, given that event A has occurred, is

\[ P(B | A) = \frac{P(A \text{ and } B)}{P(A)} \]

Example: The probability a MCC student is enrolled in an online course is 0.42. The probability a MCC student is enrolled in an on-campus class is 0.88. The probability that a MCC student is enrolled in an online and an on-campus class is 0.30.

Given: \( P(\text{Online}) = 0.42 \)
\( P(\text{On Campus}) = 0.88 \)
\( P(\text{Online and On Campus}) = 0.30 \)

Find the probability a MCC student is enrolled in an on-campus class given that they are enrolled in an online class.

\[ P(\text{On Campus} | \text{Online}) = \frac{P(\text{On Campus and Online})}{P(\text{Online})} = \frac{0.3}{0.42} = 0.714 \]

Find the probability that an on-campus student is also an online student.

\[ P(\text{Online} | \text{On Campus}) = \frac{P(\text{Online and On Campus})}{P(\text{On Campus})} = \frac{0.3}{0.88} = 0.341 \]